FIITJEE

ALL INDIA TEST SERIES

PART TEST – I

JEE (Main)-2024

TEST DATE: 18-11-2023

Time Allotted: 3 Hours **Maximum Marks: 300**

General Instructions:

- The test consists of total 90 questions.
- Each subject (PCM) has 30 questions.
- This question paper contains **Three Parts.**
- **Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics.
- Each part has only two sections: **Section-A and Section-B**.
- **Section – A** : Attempt all questions.
- **Section – B :** Do any five questions out of 10 questions.

Section-A (01 – 20, 31 – 50, 61 – 80) *contains 60 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

Section-B (21 – 30, 51 – 60, 81 – 90) *contains 30 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

Physics

PART – A

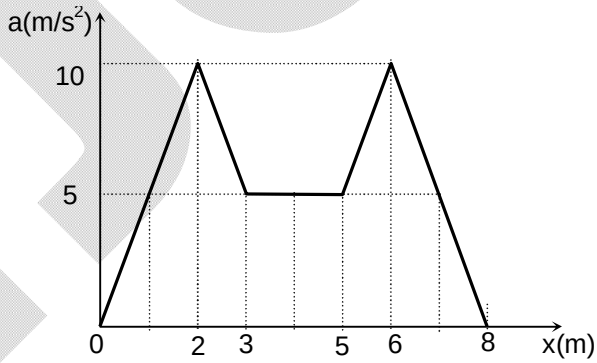
SECTION – A
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

1. The density of a material in SI units is 256 kg/m^3. In certain units in which the unit of length is 25 cm and the unit of mass is 50 g. Find the numerical value of density of the material.
(A) 40 (B) 80
(C) 128 (D) 512

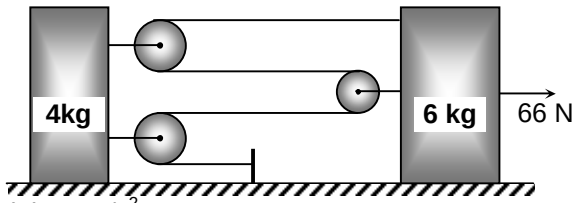
2. A freight train of length 180 m travels at speed of 40 km/h. Another passenger train of length 60 m travels at a speed of 90 km/h. The ratio of the time taken by the passenger train to completely cross the freight train when (i) they are moving in same direction, and (ii) in the opposite directions is
(A) $\frac{13}{5}$ (B) $\frac{5}{13}$
(C) $\frac{11}{5}$ (D) $\frac{13}{11}$

3. A car is moving on a horizontal road whose acceleration versus position graph is drawn below. At the initial moment of time when particle was at x = 0, its speed was $\sqrt{10}$ m/s. Find the speed of the particle when it reaches at x = 8 m.



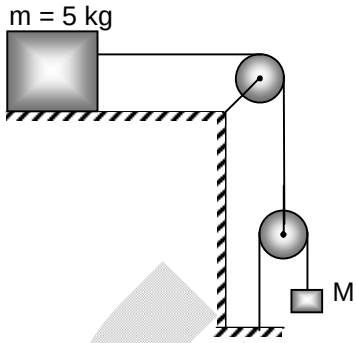
(A) $\sqrt{84}$ m/s (B) $\sqrt{110}$ m/s
(C) 10 m/s (D) 11 m/s

4. Consider the situation shown in the figure. A force F = 66 N is applied on 6 kg block. The pulleys and string are massless. There is no friction anywhere. Find the difference in acceleration of 6 kg block and 4 kg block.



(A) 3 m/s^2
(B) –3 m/s^2
(C) 2 m/s^2
(D) –2 m/s^2

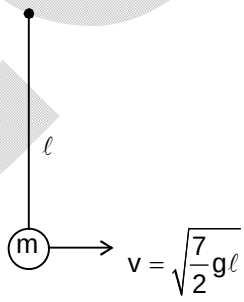
5. In the given arrangement m = 5 kg and coefficient of friction between 5 kg and surface is $\mu = 0.4$. Find the maximum value of mass 'M' of hanging block for which m = 5 kg is at rest.



(A) 1 kg (B) 2 kg
(C) 3 kg (D) 4 kg

6. A force exerts an impulse 10 N-s on an object changing its velocity from 3 m/s to 6 m/s in the direction of impulse. The force and object motion are along the straight line. Find the work done by the force.
(A) 45 J (B) 40 J
(C) 15 J (D) 60 J

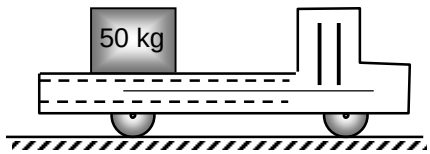
7. A small ball is given a velocity $\sqrt{\frac{7}{2}g\ell}$ at the lowest point of vertical circle.
If point of suspension is at rest then, find the maximum height reached by the ball from the point of projection as shown in the figure.



(A) $\frac{5\ell}{4}$ (B) $\frac{27\ell}{18}$
(C) $\frac{27\ell}{16}$ (D) $\frac{27\ell}{14}$

8. Find the x-coordinate of centre of mass of a uniform plate bounded by the parabola $y = x^2$ and $x = y^2$, where x and y are in meter
(A) $\frac{1}{2}$m (B) $\frac{3}{20}$m
(C) $\frac{9}{10}$m (D) $\frac{9}{20}$m

9. A box of dimensions 1.5 m $\times$ 0.75 m $\times$ 0.75 m and mass 50 kg rests on a truck with smaller face down. If accleration of truck is gradually increased, what is the minimum value of coefficient of friction for which the box topples before it slides? (Take g = 10 m/s^2)



(A) 0.4 (B) 0.5
(C) 0.6 (D) 0.3

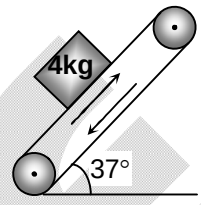
10. A cylinder is given an initial angular velocity ω and is placed on a rough horizontal surface consider the following statement(s):
(i) velocity when pure rolling starts
(ii) time until pure rolling begins
(iii) the displacement of the cylinder until pure rolling begins
(iv) the work done by the force of friction
Which of the following quantities will not depend on the coefficient of friction?
(A) (i), (iv) (B) (i), (iii), (iv)
(C) (ii), (iv) (D) (i), (iii)

11. A force $\vec{F} = \frac{-k(y\hat{i} - x\hat{j})}{\sqrt{x^2 + y^2}}N$ is applied on a block, whose position is P(x, y). The block moves on a circualr path $x^2 + y^2 = R^2$. Find the work done by the force $\vec{F}$ in two complete revolutions.

(A) $k\pi R$ (B) $2\,k\pi R$

(C) zero (D) $4\,k\pi R$

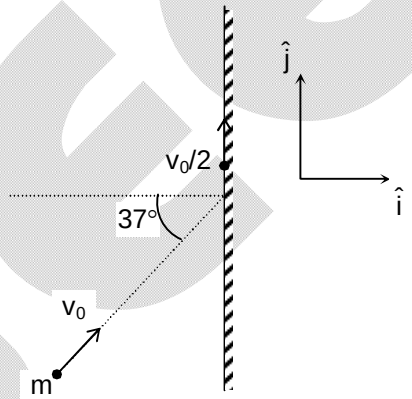
12. A block of mass 4 kg is stationary with respect to conveyor belt that is accelerating with 2 m/s^2 upwards at an angle of 37° as shown in the figure. What can be the coefficient of static friction between the block and the belt? (take g = 10 m/s^2)



(A) 0.6 (B) 0.8

(C) 1.2 (D) 0.5

13. A ball collides with a wall and after collision moves parallel to the wall as shown in the figure. The impulse acting on the ball during the collision is



(A) $mv_0\left(-\frac{4}{5}\hat{i} - \frac{1}{10}\hat{j}\right)$

(B) $mv_0\left(-\frac{4}{5}\hat{i} - \frac{1}{5}\hat{j}\right)$

(C) $mv_0\left(-\frac{3}{5}\hat{i} - \frac{1}{10}\hat{j}\right)$

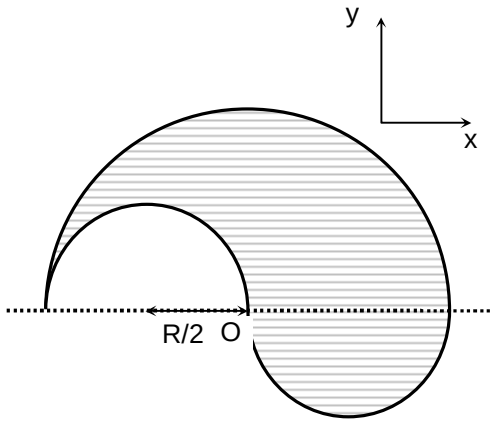
(D) $mv_0\left(-\frac{3}{5}\hat{i} - \frac{1}{5}\hat{j}\right)$

14. A person is sitting on a cart which is accelerating with $10\sqrt{3}$ m/s^2. At a certain instant the person throws a ball vertically upwards with velocity $20\sqrt{3}$ m/s with respect to cart. Find the minimum radius of curvature in meter as observed by the person throwing the ball (take g = 10 m/s^2)

(A) 30 m (B) $30\sqrt{2}$ m

(C) 45 m (D) $45\sqrt{2}$ m

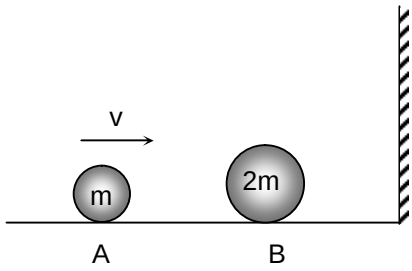
15. From a uniform semicircular thin disc of radius R, a semicircular portion of radius R/2 is removed and placed in a new position as shown in figure. Taking O as origin and x and y axis as shown in the figure, centre of mass of new setup is at



(A) $\left(\frac{R}{2}, \frac{3R}{2\pi}\right)$ (B) $\left(\frac{R}{2}, \frac{R}{\pi}\right)$

(C) $\left(\frac{R}{4}, \frac{3R}{2\pi}\right)$ (D) $\left(\frac{R}{4}, \frac{R}{\pi}\right)$

16. The two balls A and B as shown in the figure are of masses m and 2 m, respecitvely. Ball A moves with velocity 'v' towards right while B is at rest. The wall at the extreme right is fixed. Coefficient of restitution between balls is $\frac{1}{2}$ and between ball and wall is 1. What is the speed of A after all possible collisions? (assume there is no friction anywhere)



(A) v/2 (B) v/4
(C) v/8 (D) v/6

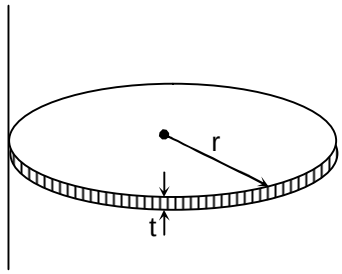
17. If E, L, M and G denote the quantities as energy, angular momentum, mass and universal gravitational constant respectively, then the dimensions of A in the formula $A = EL^2M^{-5}G^{-2}$ are:

(A) $\left[ML^0T^0\right]$ (B) $\left[M^{-1}L^2T^{-2}\right]$
(C) $\left[ML^2T^0\right]$ (D) $\left[M^0L^0T^0\right]$

18. A body of mass 2 kg begins to move under the action of a time dependent force $\vec{F} = (2t\hat{i} + 3t^2\hat{j})N$, where $\hat{i}$ and $\hat{j}$ are unit vectors along x-axis and y-axis. What is the power developed by above force at t = 2 sec?

(A) 40 W (B) 46 W
(C) 56 W (D) 80 W

19. A solid sphere of radius R has moment of inertia I about its diameter. It is melted into a disc of radius r and thickness t. If its moment of inertia about the tangential axis (which is perpendicular to plane of disc) is also equal to I, then value of t and r are respectively.



(A) 5R, $\frac{2R}{15}$ (B) 5R, $\frac{2R}{\sqrt{15}}$
(C) 2R, $\frac{2R}{15}$ (D) 2R, $\frac{2R}{\sqrt{15}}$

20. A balloon is ascending upward with acceleration 1 m/s^2. Two stones are dropped from it at an inverval of 2 sec. Find the distance between stones 1.5 sec after second stone is released. (Take g = 10 m/s^2)

(A) 40 m (B) 45 m
(C) 50 m (D) 55 m

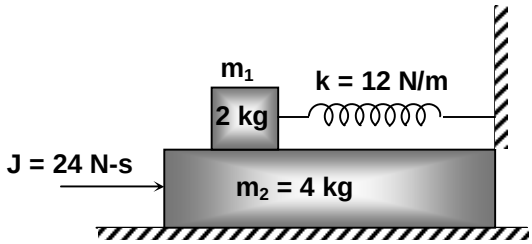
SECTION – B

(Numerical Answer Type)

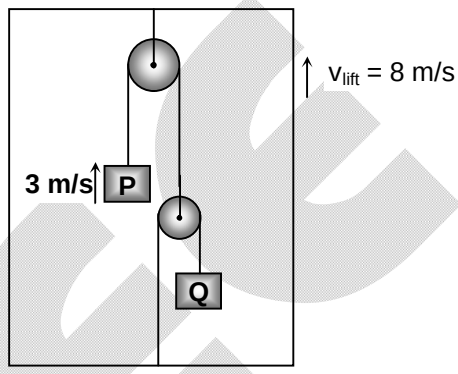
This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

21. A physical quantity p is given as $p = \frac{a^2b}{(4-c)^3}$, where a = (2 ± 0.01) in SI units, b = (4 ± 0.02) in SI units and c = (2 ± 0.01) in SI units. Find percentage error in calculation of p.

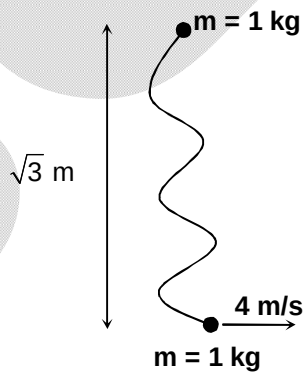
22. A block of mass m_1 = 2 kg is placed on a plank of mass m_2 = 4 kg, which is placed on horizontal frictionless surface. There is no friction between the block and the plank. Block and plank are connected by a spring of force constant 12 N/m as shown in the figure. An impulse J = 24 N-s is applied on the plank. Find the maximum elongation of spring in cm.



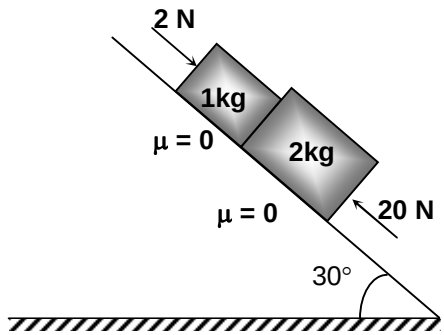
23. Consider the situation shown in the figure. At any instant velocity of lift is 8 m/s. Velocity of block P relative to lift is 3 m/s. Find the speed of Q (in m/s) relative to ground at that instant.



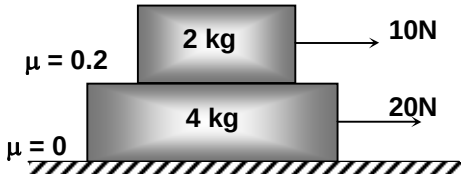
24. Two particles each of mass 1 kg are connected by a string of length 2 m and placed on a smooth horizontal plane. Initially they are separted by $\sqrt{3}$ m. If one of the particle is given velocity 4 m/s as shown in the figure. Find the tension in string in newton just after it becomes taut.



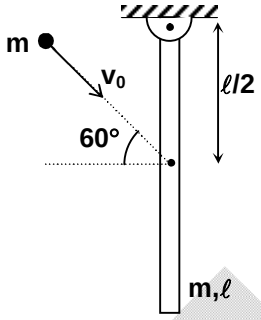
25. Find the normal reaction force between the blocks in newton. (take g = 10 m/s^2)



26. If there is no relative motion between the two blocks, find force of friction in newton between the blocks.

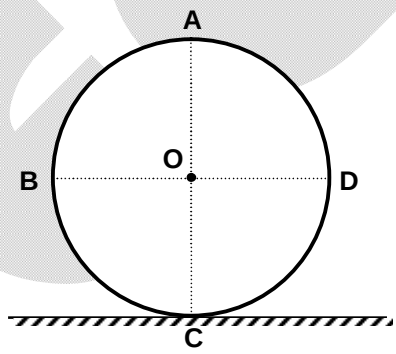


27. A thin rod of mass m = 1 kg and length ℓ = 3m is hinged to a ceiling end is free to rotate in a vertical plane. A particle of mass m = 1 kg moving with speed v_0 strikes as shown in figure and gets stick with the rod. The value of v_0 in (cm/s) for which the rod becomes horizontal after collision is (Take g = 10 m/s^2, $\sqrt{35}$ = 5.9)

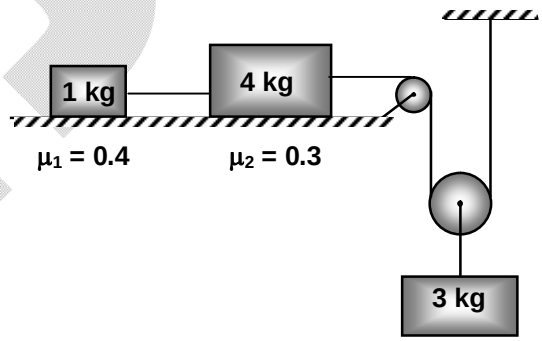


28. If $\vec{A} = 3\hat{i} + \sqrt{3}\hat{j} + 2\hat{k}$ and $\vec{B} = 4\hat{i} + \sqrt{3}\hat{j} + 2.5\hat{k}$ is given. If unit vector in the direction of $\vec{A} \times \vec{B}$ is $\alpha(\sqrt{3}\hat{i} + \hat{j} - 2\sqrt{3}\hat{k})$. Find the value of 100α.

29. A wheel rolls on a plane surface without slipping. At a certain instant velcoity and accelertion of centre 'O' are 1 m/s and 3 m/s^2. If radius of wheel is 0.5 m. BD is horizontal and AC is vertical. If at that instant acceleration of point A is $\sqrt{a}$ m/s^2, then find the value of a



30. Find the tension in the string (in N) connecting 1 kg and 4 kg block as shown in the figure. (Initially strings are unstretched and g = 10 m/s^2)



Chemistry

PART – B

SECTION – A
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

31. The sum of stoichiometric coefficients of all the reactants in the balanced chemical equation of $CoCl_2 + Na_2O_2 + NaOH + H_2O \longrightarrow Co(OH)_3 + NaCl$ is

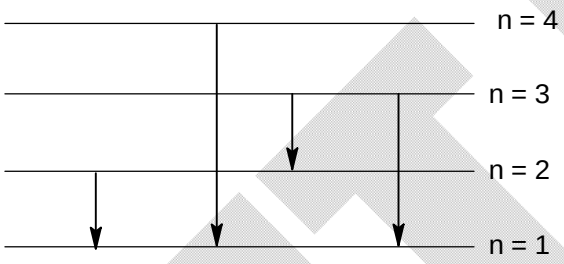
(A) 6 (B) 8
(C) 7 (D) 5

32. For the reaction $X(g) \longrightarrow 2Y(g) + \frac{1}{2}Z(g)$

If the initial pressure of the system is 600 mm Hg and pressure at any time is 900 mm Hg, then the mole fraction of X(g) decomposed at constant volume and temperature is…..
(Assume ideal gas behaviour)

(A) 0.197 (B) 0.333
(C) 0.157 (D) 0.147

33. A hypothetical single electronic atom gives a red, green, blue and violet line spectrum as given in the following diagram



What is the jump of electron corresponding to red line

(A) $3 \rightarrow 1$ (B) $2 \rightarrow 1$
(C) $4 \rightarrow 1$ (D) $3 \rightarrow 2$

34. Which of the following is NOT correct for NO_2, $\overset{+}{N}O_2$ and $\overset{-}{N}O_2$

(A) NO_2 is paramagnetic in nature.

(B) $\overset{+}{N}O_2$ is linear, NO_2 is bent and bond angle is less than 120^o.

(C) $\overset{+}{N}O_2$ has the strongest bonds among the given species.

(D) $\overset{-}{N}O_2$ has the bond order of 1.5.

35. The shape with respect to central atom of the following molecules are in the order $N(SiH_3)_3$, $N(CH_3)_3$, $P(SiH_3)_3$

(A) Planar, Pyramidal, Planar
(B) Planar, Pyramidal, Pyramidal
(C) Pyramidal, Pyramidal, Tetrahedral
(D) Pyramidal, Planar, Pyramidal

36. The INCORRECT statement among the following about alkaline earth metals
(A) Solubility of sulfates decreases down the group.
(B) Thermal stability of carbonates increases down the group.
(C) $(BeCl_2)_n$ has 3c – 4e bonds.
(D) Solubility of hydroxides decreases down the group.

37. Consider the following transformations:
(P) $O(g) + e^- \longrightarrow O^-(g)$, $\Delta H_1^{\ominus}$
(Q) $F(g) + e^- \longrightarrow F^-(g)$, $\Delta H_2^{\ominus}$
(R) $Cl(g) + e^- \longrightarrow Cl^-(g)$, $\Delta H_3^{\ominus}$
(S) $O^-(g) + e^- \longrightarrow O^{--}(g)$, $\Delta H_4^{\ominus}$
Now select the INCORRECT statement among the following
(A) $\Delta H_3^{\ominus}$ is more negative than $\Delta H_1^{\ominus}$ and $\Delta H_2^{\ominus}$
(B) $\Delta H_1^{\ominus}$ is less negative than $\Delta H_2^{\ominus}$
(C) $\Delta H_1^{\ominus}, \Delta H_2^{\ominus}, \Delta H_3^{\ominus}$ are negative while $\Delta H_4^{\ominus}$ is positive
(D) $\Delta H_1^{\ominus}, \Delta H_3^{\ominus}$ are negative while $\Delta H_2^{\ominus}$ and $\Delta H_4^{\ominus}$ are positive

38. $M(OH)_4(s)$ is involved in the following two equilibriums as follows

$$M(OH)_4(s) \rightleftharpoons M^{4+}(aq) + 4OH^-(aq)$$
$$K_{sp} = 10^{-20}$$
$$M(OH)_4(s) + 2OH^-(aq) \rightleftharpoons \left[M(OH)_6\right]^{2-}(aq)$$
$$K_C = 2 \times 10^{-2}$$

Then the pH at which solubility is minimum?
(A) 10 (B) 11
(C) 12 (D) 13

39. A certain volume of 0.2 M benzoic acid (pKa = 4.2) solution is added to a 0.4 M potassium benzoate solution and the resultant volume was found to be 600 mL. If pH of the solution is found to be 4.5 then find the volume of benzoic acid solution added. (log2 = 0.3)
(A) 200 mL (B) 400 mL
(C) 300 mL (D) 100 mL

40. A_3B_4 is a sparingly soluble salt of molar mass is x g mol^{-1} and solubility of this salt in water is y g L^{-1}, then the ratio of concentration of $A^{4+}(aq)$ to K_{sp} of the salt is
(A) $\frac{1}{2304}\frac{x^6}{y^6}$ (B) $\frac{1}{256}\frac{x^6}{y^6}$
(C) $\frac{1}{768}\frac{y^6}{x^6}$ (D) $\frac{1}{1012}\frac{x^6}{y^6}$

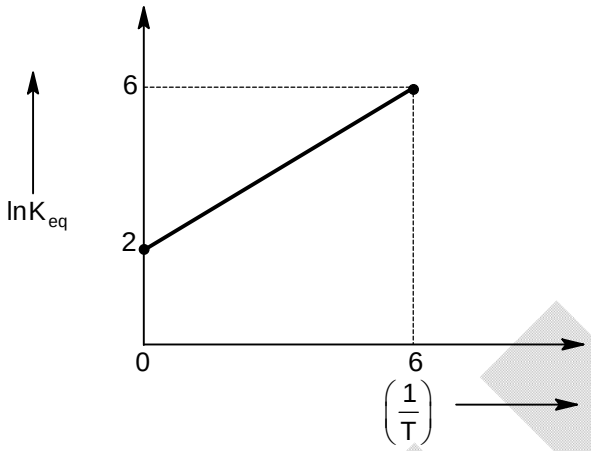
41. 2 mol of $X_2(g)$ contained in a flask of volume 2 L and at 27°C is allowed to react with 2 mol of $Y_2(g)$ contained in another flask of volume 6 L and at 27°C, after connecting the two flasks by a narrow glass tube, the reaction proceeds as follows:

$$X_2(g) + Y_2(g) \rightleftharpoons 2XY(g) \qquad (K_c = 16)$$

The concentration of XY (g) at equilibrium is

(A) 0.33 M (B) 0.44 M
(C) 0.66 M (D) 1.00 M

42. Consider the following plot of $\ln K_{eq}$ vs. $1/T$ (in appropriate units) for a reaction



Select the CORRECT statement with respect to the plot of the reaction

(A) Always non-spontaneous
(B) Spontaneous at high temperature but non-spontaneous at low temperature
(C) Spontaneous at low temperature but non-spontaneous at high temperature
(D) Always spontaneous

43. Consider the following equilibriums

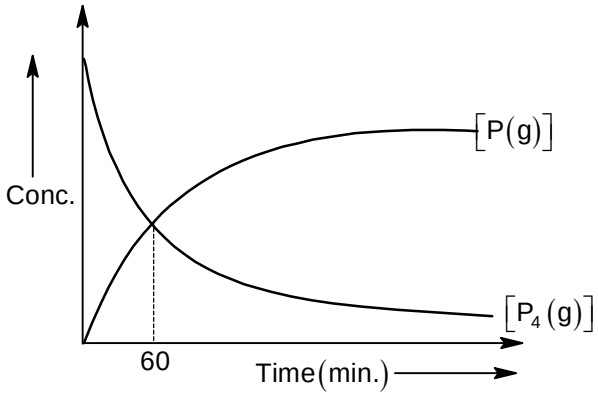
$$N_2O_3(g) \rightleftharpoons NO_2(g) + NO(g) \qquad (K_{P_1} = 1\,\text{bar})$$

$$2NO_2(g) \rightleftharpoons N_2O_4(g) \qquad (K_{P_2} = ?)$$

Reaction starts with pure $N_2O_3(g)$ having pressure 4 bar and at equilibrium the pressure of NO(g) is found to be 2.0 bar. The value of K_{P_2} is

(A) 0.60 (B) 0.25
(C) 0.50 (D) 0.40

44. Consider the following 1st order decomposition reaction $P_4(g) \longrightarrow 4P(g)$, $[\log 2 = 0.30]$



Select the INCORRECT statement regarding the reaction
(A) At t = 60 min, only 20 % reaction is complete
(B) $t_{1/2}$ of the reaction is 180 min
(C) Rate of the reaction decreases linearly with time
(D) Time for intersection of two curves is independent of initial concentration of $P_4(g)$

45. The reaction $X(g) + 2Y(g) \longrightarrow Z(g) + W(g)$ is an elementary reaction. In an experiment partial pressures of X(g) and Y(g) are 0.60 and 0.80 atm respectively. Calculate the ratio of rate of reaction (when $P_Y = 0.20$ atm.) to the initial rate of reaction

(A) $\frac{1}{16}$ (B) $\frac{1}{24}$
(C) $\frac{1}{32}$ (D) $\frac{1}{48}$

46. The wavelength of K_α line for an element of atomic number 57 is λ. Then the wavelength of K_α line for the element having atomic number 29 is
(A) λ (B) 2λ
(C) 4λ (D) 3λ

47. The compressibility factor (Z) for 1 mol of a gas obeying van der Waal's equation at Boyle's temperature (T_B) is
(A) $1+\frac{b^2}{V_m(V_m-b)}$ (B) $1+\frac{b^2}{V_{m^2}}$
(C) $1-\frac{b}{V_m}$ (D) $1-\frac{b^2}{V_{m^2}}$

48. Choose the INCORRECT statement among the following
(A) $\Delta_{ion}H_1^{\ominus}$ of sodium is less than that of magnesium
(B) $\Delta_{ion}H_3^{\ominus}$ of aluminium is less than that of magnesium
(C) $\Delta_{ion}H_1^{\ominus}$ of magnesium is greater than that of aluminium
(D) $\Delta_{ion}H_2^{\ominus}$ of nitrogen is greater than that of oxygen

49. The strength of backbonding is MAXIMUM among the following compounds is ….
(A) PF_3 (B) BF_3
(C) $(SiH_3)_3N$ (D) $(SiH_3)_2O$

50. Select the INCORRECT statement about the diagonal relationship between Al and Be
(A) Both BeO and Al_2O_3 are amphoteric in nature
(B) Both Be_2C and Al_4C_3 on hydrolysis yields the same gas
(C) Both Be and Al can form complexes
(D) Both BeH_2 (Solid) and AlH_3(Solid) crystallizes in the same structure

SECTION – B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

51. A hydrogen like atom of atomic number Z is in the excited state of quantum number '2n'. It can emit a maximum energy photon of 204 eV. If it makes a transition to a quantum state 'n' a photon of energy 40.8 eV emitted. Then the value of 'Z' is…..

52. Consider the following reaction is of first order kinetics
$2X(g) \longrightarrow 4Y(g) + Z(g) + W(\ell)$
After 30 min from the start of the reaction total pressures was found to be 645 mm and after a long time, the total pressure is 825 mm. The vapour pressure of $W(\ell)$ at this temperature is 25 mm. The partial pressure of (Y) after 60 min is …. (mm).

53. 10 mol each of P(g), Q(g) and R(g) are added in a 20 L flask and the following two equilibriums are observed
$P(g) + Q(g) \rightleftharpoons X(g); \quad K_c = 16 \times 10^{10}$
$P(g) + R(g) \rightleftharpoons Y(g); \quad K_c = 4 \times 10^{10}$
If at equilibrium number of mols of Q(g) are n, then the value of 15n is…..

54. The average concentration of SO_2 over a city's atmosphere on a certain day is 10 ppm., when the temperature is 298 K. If the solubility of SO_2 in water at 298 K is $1.28\,gL^{-1}$ and pK_{a_1} of H_2SO_3 is 2.0, then the pH of rain on that day is……

55. The equation of state for a certain gas is $\left(P + \frac{x}{V_m}\right)(V_m - y) = RT$, where x, y are constants distinct from zero. Check whether the gas has a critical point or not. If the answer is yes then mark 11 and if answer is no mark 33.

56. The number of species among the following which are able to show phenomenon of backbonding.
BF_3, Me_2O, PF_3, POF_3, $(SiH_3)_3N$, Cl_2O, MeNCS, Me_3N, CCl_3^-, H_4SiO_4

57. Consider the following species
SF_4, BrF_3, XeF_2, IF_5, SbF_4^-, SF_5^-, CH_3^+, SeF_3^+, $\overset{+}{P}Cl_4$, PCl_5, ICl_2^-, NO_3^-, XeO_2F_2, XeF_4
If number of planar species = x and number of non-planar species is y, then the value of $|x - y|$ is….

58. The sum total of number of (2c – 2e) and (3c – 4e) bonds present in the dimer of aluminium hydroxide is…..

59. The number of electron(s) in an element having Z = 46 whose magnetic quantum number is zero.

60. 2 mol each of potassium oxide, potassium peroxide and potassium superoxide undergoes hydrolysis separately. The number of mols of H_2O_2 produced is…..

Mathematics

PART – C

SECTION – A
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

61. Let $f(x) = \begin{cases} \left(\frac{p(x)}{10}\right)^{\frac{1}{\tan(x^2-1)}} & x \neq 1 \\ e^{\frac{3x}{3(a^2-2a)+13}} & x = 1 \end{cases}$

where p(x) is polynomial function satisfying $p(x) - p'(x) = x^2 + 2x + 1$, if f(x) is continuous at x = 1, then value of 'a' can be

(A) -1 (B) 1
(C) 2 (D) -2

62. Let g be the differentiable function satisfying $\int_0^x (x-t+1)g(t)dt = x^4 + x^2 \ \forall \ x \geq 0$, then $\int \frac{12}{g(x)+g'(x)-14}dx$ is equal to

(A) $6\ln\left|\frac{x-1}{x+1}\right| + c$ (B) $12\ln\left|\frac{x+1}{x-1}\right| + c$
(C) $\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + c$ (D) none of these

63. The value of the definite integral $\int_{-2}^{2} x^3 \ln\left(1^x + 3^x + 5^x + 15^x\right)dx$ is equal to

(A) $\frac{\ln 15}{4}$ (B) $\frac{64}{5}\ln 15$
(C) $\frac{32}{5}\ln 15$ (D) $\frac{64}{5}\ln 30$

64. Let $f(x) = (x + \log_3 x)^2 + x^2 \ \forall \ x > 0$, then derivative of $f^{-1}(x)$ w.r.t x at x = 25 is

(A) $\frac{3\ln 3}{42\ln 3 + 8}$ (B) $\frac{1}{42\ln 3 + 8}$
(C) $\frac{\ln 3}{3\ln 3 + 8}$ (D) $\frac{3\ln 3}{42\ln 3 - 8}$

65. Let $f : R \to (0, \infty)$ be such that $f(x) + \frac{e^{x^2+x^4}}{f(x)} \leq e^{x^2} + e^{x^4} \ \forall \ x > 0$, then $\lim_{x\to 1} f(x)$ is equal to

(A) e^2 (B) e^3
(C) e (D) 1/e

66. Let $I_1 = \int_0^1 \frac{e^x}{1+x}dx$ and $I_2 = \int_0^1 \frac{x^3}{e^{x^4}(2-x^4)}dx$, then $\frac{I_1}{I_2}$ is equal to

(A) $\frac{e}{4}$ (B) 4e

(C) $\frac{3}{e}$ (D) 3e

67. Given that the slope of the tangent to the curve y = f(x) at point (x, y) is $\frac{6y}{x^3}$, if the curve passing through the focus of parabola $y^2 - 8x - 2y - 7 = 0$, then its equation is

(A) $x^2 \ln|y| = 3(x^2 + 1)$ (B) $x \ln y = 3(x - 1)$

(C) $x^2 \ln|y| = 3(x - 1)$ (D) $x^2 \ln|y| = 3(x^2 - 1)$

68. The area of smaller region bounded by the curve $9x^2 + 4y^2 - 36x + 16y + 16 = 0$ and the line $3x + 2y = 8$ is

(A) $\frac{3}{2}(\pi + 2)$ (B) $3(\pi - 2)$

(C) $\frac{3}{4}(\pi - 2)$ (D) $\frac{3}{2}(\pi - 2)$

69. The area of the region bounded by curves $y = \frac{x^2}{a}$ and $y = x - ax^2$ is maximum, then value of a is

(A) 1 (B) 2

(C) 3/2 (D) none of these

70. If $\lim_{x\to 0} \frac{\sin 2x - \cos 3x + ke^{2x} - \frac{x^3}{3}}{x^n} = L$ (where L is non zero finite number), then

(A) n = 1, k = 2 (B) n = 1, L = 4

(C) k = 1, L = 3 (D) n = 2, L = 4

71. Let the function f(x) satisfying $x^2f'(x) + 2xf(x) = xe^x$ and f(1) = 2, then

(A) domain of f(x) is $x \in R - \{0\}$ (B) f(x) is discontinuous at x = 1

(C) f(x) is continuous at x = 0 (D) f is not differentiable at x = 1

72. Let $f(x) = x^3 + 3x + 1$, if g(x) is the inverse of f(x), then $\int_1^5 g(x)dx$

(A) 9/4 (B) 7/4

(C) 7/2 (D) 11/4

73. The tangent to the curve $y = 2xe^{x^2-x+1}$ passing through the point (1, 2e) also passes through the point

(A) (2, 5) (B) (5, 2e)

(C) (3, 2e) (D) (2, 6e)

74. Let f(x) be differentiable function on the interval $(0, \infty)$ such that f(1) = 1 and $\lim\limits_{t\to x}\left(\dfrac{t^3f(x)-x^3f(t)}{t^2-x^2}\right)=\dfrac{1}{2}$ $\forall$ x > 0, then the area bounded by y = f(x), ordinate x = 1, x = 2 and x-axis is

(A) $\dfrac{45}{16}+\dfrac{1}{4}\ln 2$ (B) $\dfrac{45}{16}$

(C) $\dfrac{99}{32}+\dfrac{1}{4}\ln 2$ (D) $\dfrac{99}{16}+\dfrac{1}{4}\ln 2$

75. Let y = f(x) be differentiable function satisfying $\int\limits_2^x f(t)dt+2x=\dfrac{x^2}{2}+\int\limits_x^2 t^2f(t)dt$, then range of f(x) $\forall$ x $\in$ [5, 7] is

(A) $\left(-\dfrac{2}{17},\dfrac{1}{10}\right)$ (B) $\left[\dfrac{1}{17},\dfrac{2}{17}\right)$

(C) $\left(\dfrac{2}{17},\dfrac{3}{26}\right]$ (D) $\left[\dfrac{1}{10},\dfrac{3}{26}\right]$

76. Let I = $\int\limits_0^1 \dfrac{5x^4\left(1+(x^5)^{2022}\right)}{(1+x^5)^{2024}}dx=\dfrac{p}{q}$ where p and q are relatively prime integer, then number of divisor of q is

(A) 16 (B) 6

(C) 10 (D) 2

77. The acute angle between tangents to the curve $f(x)=\int\limits_0^x e^{t(t-1)}\cdot(t+1)(t+2)dt$ at x = 0 and x = 1 is

(A) $\cot^{-1}\dfrac{14}{13}$ (B) $\tan^{-1}\dfrac{13}{4}$

(C) $\cot^{-1}\dfrac{13}{4}$ (D) $\tan^{-1}\dfrac{8}{13}$

78. The function f : $(-\infty, 3] \to (0, 3]$ define by $f(x) = e^{x^3-3x^2-9x+\ln 3-5}$ is

(A) many one and into (B) many one and onto

(C) one-one and onto (D) one-one and into

79. Let the function f be define by $f(x)=\begin{cases} ax^2+bx+5 & x<3 \\ bx^2+2ax & x\geq 3\end{cases}$ is differentiable $\forall$ x $\in$ R, then

(A) $a=\dfrac{25}{9},b=\dfrac{20}{9}$ (B) $a=\dfrac{20}{9},b=\dfrac{25}{9}$

(C) $a=\dfrac{4}{9},b=\dfrac{5}{9}$ (D) $a=\dfrac{5}{9},b=\dfrac{4}{9}$

80. Let f(x) be a polynomial satisfying $\lim\limits_{x\to\infty}\dfrac{x^3f(x)}{x^6+1}=2$, f(1) = 5, f(–1) = 3, f(0) = 2, then number of points of discontinuity of $g(x)=\dfrac{1}{f(x)-2-2x^2}$ in (0, 5) is equal to

(A) 3 (B) 2

(C) 1 (D) 4

SECTION – B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

81. If $\int_0^{2024} f(x)dx = 30$, then $\left[\cos^{-1}\cos\left(\frac{\sum_{r=1}^{2024}\int_0^1 f(r-1+x)dx}{6}\right)\right]$ (where [.] represent G.I.F.) is equal to

82. Let f(x) satisfying the condition $f(x) + 3f(2 - x) = 2x^3 + 3$, then value of $|4f(4)|$ is

83. Let 'a' be the number of points of discontinuity of the function $f(x) = [x^3 + x]$ is $x \in (0, 2)$, then the value of $\int_{-a}^{a}(\{x\}+\{1-x\})dx$ when [.] and {x} are greatest integer function and fractional part

84. Let $f(x) = 2x^3 - 3x^2 - 36x + 120$ have its local maxima and local minima at a, b respectively and length of tangent drawn at (a, b) to the curve $x^2 + (y - 3)^2 = 4$ is equal to

85. Number of points of non differentiability of $f(x) = [5\sin x - 3]$ $\forall$ $x \in (0, \pi)$ is 'k'
and $a = \lim_{x\to 0}\frac{x+\cos 2x - e^x}{x^2}$, then the value of $k^{-\frac{2a}{5}}$ is

86. If $f : R \to (0, \infty)$ be a function satisfying the condition $f(x + y) - f(x - y) = \frac{f(x)\,(f(3y)-f(y))}{f(2y)}$ $\forall$ y, $x \in R$, $f'(0) = \ln 3$, $f(0) = 1$ then f(2) is equal to

87. Let $f(x) = \frac{x^7}{7}+\frac{x^5}{5}+\frac{x^3}{3}+x+1$ and g(x) is inverse of f(x), then $|g''(1) - 3|$ is equal to

88. Let $f(x) = \frac{ax^3}{3}+\frac{bx^2}{2}+5x+7$ is an injective function $\forall$ $x \in R$, where a, b $\in$ {1, 2, …… 9}, then number of ordered pair (a, b) is

89. Number of solution of $e^{x-[x]} = |\sin|x|| + 1$ in interval $x \in [0, \pi]$ is equal to

90. If $(f(x) - 3)(x^2 - x + 1)^2 - (f(x) + 3)(x^4 + x^2 + 1) = 0$ $\forall$ $x \in R^+$, then minimum value of $|f(x)|$ is equal to