FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2024
PART TEST – I
PAPER –1
TEST DATE: 19-11-2023

Time Allotted: 3 Hours **Maximum Marks: 180**

General Instructions:

- The test consists of total 51 questions.
- Each subject (PCM) has 17 questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Two Sections**: **Section-A & Section-B**.

Section – A *(01 –03, 18 – 20, 35 – 37):* This section contains **NINE (9)** questions. Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).

Section – A *(04 – 07, 21 – 24, 38 – 41)*: This section contains **TWELVE (12)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.

Section – A *(08 – 11, 25 – 28, 42 – 45):* This section contains **TWELVE (12)** Matching List Type Questions. Each question has **FOUR** statements in **List-I** entries (P), (Q), (R) and (S) and **FIVE** statements in **List-II** entries (1), (2), (3), (4) and (5). The codes for lists have choices (A), (B), (C), (D) out of which, **ONLY ONE** of these four options is correct answer.

Section – B *(12 – 17, 29 – 34, 46 – 51):* This section contains **EIGHTEEN (18)** numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.

MARKING SCHEME

Section – A (One or More than One Correct): Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If only (all) the correct option(s) is (are) chosen;
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen;
Partial marks	:	+2	If three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–2	In all other cases.

Section – A (Single Correct): Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If ONLY the correct option is chosen.
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–1	In all other cases.

Section – B: Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If ONLY the correct numerical value is entered at the designated place;
Zero Marks	:	0	In all other cases.

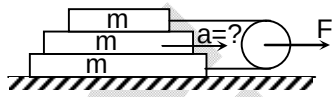
Physics

PART – I

SECTION – A
(One or More than one correct type)

*This section contains **THREE (03)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

1. A block of mass m lies on a slippery horizontal surface. On top of it lies another block of mass m, and on top of that another block of mass m. A thread that connects the first and the third block has been extended around a weightless pulley. The threads are horizontal and the pulley is being pulled by a force F. The coefficient of friction between the blocks is μ. The acceleration of the second block is :



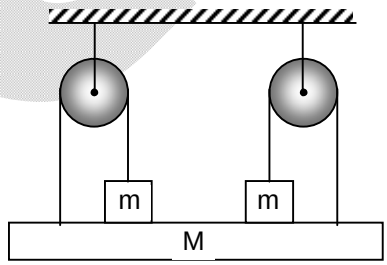
(A) $\frac{F}{3m}$, if $\frac{F}{\mu mg} = 5$

(B) $\frac{F}{4m} + \frac{\mu g}{2}$, if $\frac{F}{\mu mg} = 8$

(C) $3\mu g$, if $\frac{F}{\mu mg} = 12$

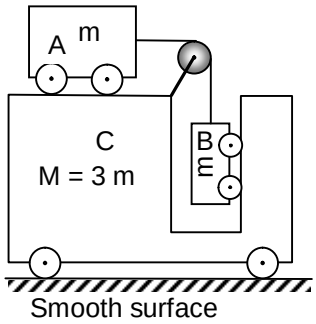
(D) $3\mu g$, if $\frac{F}{\mu mg} = 14$

2. A board of mass M was fixed symmetrically as shown in the figure. The masses of the fixed pulleys and the ropes, as well as friction in the shaft are negligible. (The objects of mass m are not glued to the board of mass M.) For what combinations of m and M will the system be in equilibrium?



(A) m=1 kg , M = 2 kg
(B) m=2 kg , M = 2 kg
(C) m=2 kg , M = 8 kg
(D) m=8 kg , M = 2 kg

3. In the adjoining figure, all surfaces are smooth. Then choose the correct option(s).



(A) the acceleration of the trolley C is g/9

(B) the acceleration of the body A relative to ground is $\frac{4g}{9}$

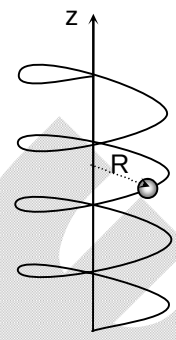
(C) the tension in the string is $\frac{2mg}{9}$

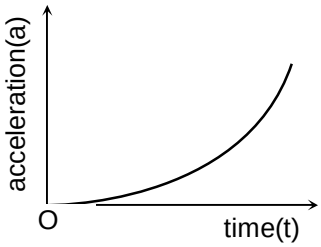
(D) the tension in the string is $\frac{4mg}{9}$

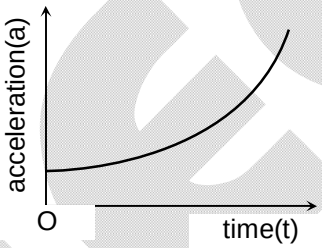
SECTION – A
(One Options Correct Type)

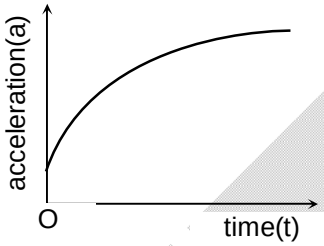
*This section contains **FOUR (04)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.*

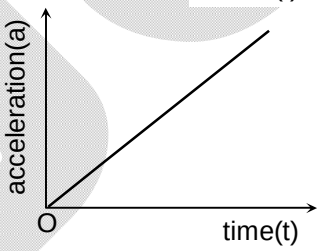
4. A bead slides on a wire with the shape of a helix as shown in the figure, whose symmetry axis is oriented vertically. Which of the following graphs best represents the acceleration of the bead as a function of time?



(A)


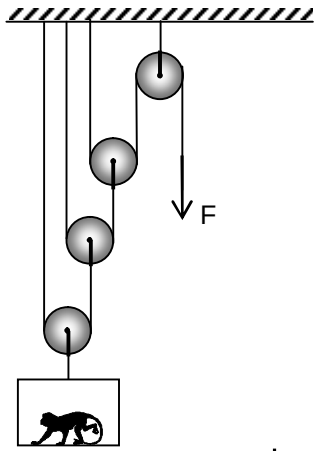
(B)


(C)


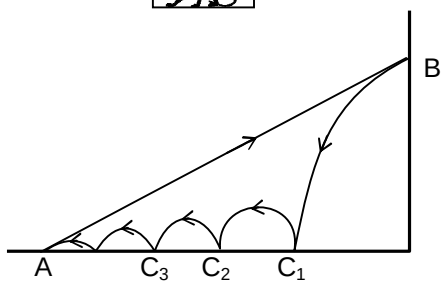
(D)


5. The mass of each pulley in the pulley system shown in the figure is 1 kg. What is the magnitude of the force F with which the monkey in the cage can be kept in equilibrium? The total mass of the monkey and the cage is 9 kg. ($g = 10\ m/s^2$)

(A) 10 N (B) 20 N
(C) 30 N (D) 40 N



6. A ball was projected from point A and it hits a vertical wall at point B horizontally. From point B the ball goes to points C_1, C_2, …, and finally gets back to point A . The coefficient of restitution is e at any point. Find the possible value of e, if the ball does not bounce up at all when it gets back to point A.



(A) $e = \sqrt{2} - 1$ (B) $e = \sqrt{3} - 1$

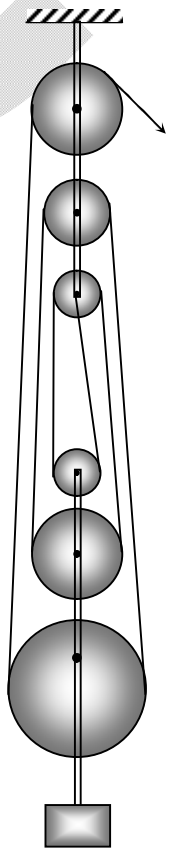
(C) $e = \frac{\sqrt{3} - 1}{2}$ (D) $e = \frac{1}{\sqrt{2}}$

7. How many of the following statements about an action-reaction force pair is/are true?
   I. The sum of the impulses due to the two forces is always zero.
   II. The sum of the torques due to the two forces is always zero.
   III. The sum of the torques due to the two forces depends on the choice of reference point with respect to which torques are measured.
   IV. The sum of the works done by the two forces is always zero.
   (A) (I) and (IV) (B) (I) and (II)
   (C) (I) and (III) (D) (II) and (IV)

SECTION – A
(Matching List Type)

*This section contains **FOUR (04)** Matching List Type Questions. Each question has FOUR statements in **List-I** entries (P), (Q), (R) and (S) and FIVE statements in **List-II** entries (1), (2), (3), (4) and (5). The codes for lists have choices (A), (B), (C), (D) out of which **ONLY ONE** of these four options is correct answer.*

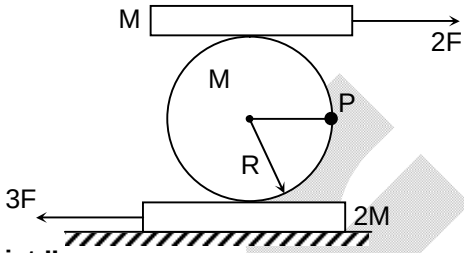
8. In the pulley system shown in the *figure* the fixed pulley at the top has a radius of 15 cm, while the radius of the movable pulley at the bottom is 25 cm. Each of the movable pulleys turns 15 whole revolutions in a minute, and the rotational speeds of the fixed pulleys are also equal. (The threads between the pulleys can be considered vertical.). Pulleys are numbered starting from top as P_1, P_2, P_3, P_4, P_5, P_6.There is no slipping of string on any pulley. Match each entry in **List-I** to the correct entries in **List-II.**



List –I		List –II	
(P)	Radius of P_2	(1)	75 cm
(Q)	Radius of P_3	(2)	5 cm
(R)	Radius of P_4	(3)	10 cm
(S)	Radius of P_5	(4)	15 cm
		(5)	100 cm

The correct option is:
(A) (P) → (3) (Q) → (2) (R) → (2) (S) → (4)
(B) (P) → (3) (Q) → (2) (R) → (5) (S) → (4)
(C) (P) → (2) (Q) → (1) (R) → (4) (S) → (5)
(D) (P) → (2) (Q) → (1) (R) → (1) (S) → (3)

9. A cylinder is sandwiched between two planks as shown in the figure. Two constant horizontal forces 2F and 3F are applied on the planks at t = 0 as shown. There is no slipping at the top and bottom of the cylinder. Horizontal surface below the plank is smooth.

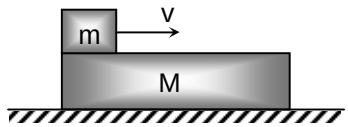


Match each entry in **List-I** to the correct entries in **List-II.**

List –I		List –II	
(P)	Acceleration of Centre of mass of cylinder at t = 0	(1)	$\frac{20F}{13M} < a < \frac{21F}{13M}$
(Q)	Acceleration of top plank at t = 0	(2)	$\frac{19F}{13M} < a < \frac{20F}{13M}$
(R)	Acceleration of bottom plank at t = 0	(3)	$\frac{F}{13M} < a < \frac{2F}{13M}$
(S)	Acceleration of point P of cylinder at t = 0	(4)	$\frac{17F}{13M} < a < \frac{18F}{13M}$
		(5)	$a = \frac{8F}{3M}$

The correct option is:
(A) (P) → (3) (Q) → (1) (R) → (4) (S) → (2)
(B) (P) → (3) (Q) → (2) (R) → (5) (S) → (4)
(C) (P) → (2) (Q) → (1) (R) → (4) (S) → (5)
(D) (P) → (2) (Q) → (1) (R) → (1) (S) → (3)

10. As shown in the figure below, a uniform rectangular block of mass M is resting on a smooth horizontal surface. A small cube with mass m and negligible length and width is on the top of the rectangular block. At time t = 0, the cube is imparted an intial velocity v towards right relative to ground. It finally stops at half the length of the rectangular block. It is known that the coefficient of kinetic friction between the small cube and the rectangular block is μ and the gravitational acceleration is g. Take M = m = 1kg, μ = 0.1, v = 2 m/s, g=10 m/s^2.



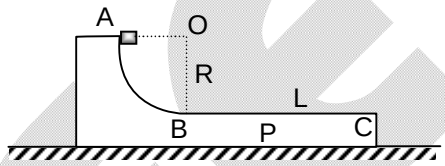
Match each entry in **List-I** to the correct entries in **List-II.**
(All the entries of List-II are in SI units)

List –I		List –II	
(P)	What is the time t_0 for the small cube to stop on the rectangular block?	(1)	0.5
(Q)	From time t = 0 to t_0, what is the distance travelled by the small cube relative to ground?	(2)	1.5

(R)	From time t = 0 to t_0, what is the distance travelled by the rectangular block relative to ground?	(3)	1
(S)	What is the length of the rectangular block?	(4)	2
		(5)	2.5

The correct option is:
(A) (P) → (3) (Q) → (2) (R) → (5) (S) → (4)
(B) (P) → (2) (Q) → (1) (R) → (4) (S) → (5)
(C) (P) → (3) (Q) → (2) (R) → (1) (S) → (4)
(D) (P) → (2) (Q) → (1) (R) → (1) (S) → (3)

11. On a smooth horizontal ground surface there is a big block P of mass M = 1 kg. Its AB section is a quarter circle of radius R= 0.1 m, while its BC section is a horizontal surface of length L = 2 m. A small cube Q of mass m = 1 kg is released from the top of the arc from rest and it slides down. When it reaches point B at the bottom of the arc its speed relative to P is v. It then continues to move forward and finally stops at point-C. Arc AB is smooth. g = 10 m/s^2.



Match each entry in **List-I** to the correct entries in **List-II.**
(All the entries of List-II are in SI units)

List –I		List –II	
(P)	The value of v when Q reaches point-B	(1)	1
(Q)	The kinetic friction coefficient between P and Q	(2)	0.05
(R)	The displacement of P relative to the ground when Q reaches point-C (assuming L>>R).	(3)	2
(S)	The horizontal displacement of the centre of mass of P and Q .	(4)	0
		(5)	0.5

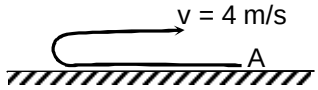
The correct option is:
(A) (P) → (3) (Q) → (2) (R) → (5) (S) → (4)
(B) (P) → (2) (Q) → (1) (R) → (4) (S) → (5)
(C) (P) → (2) (Q) → (1) (R) → (1) (S) → (3)
(D) (P) → (3) (Q) → (2) (R) → (1) (S) → (4)

SECTION – B

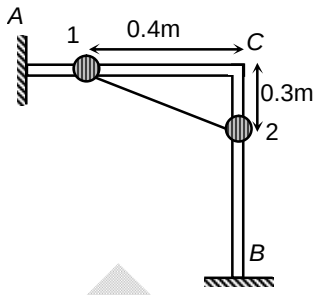
(Numerical Answer Type)

*This section contains **SIX (06)** Numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.*

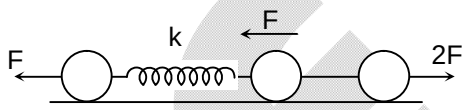
12. A long thin carpet is laid on a floor. One end of the carpet is bent back and then pulled backwards with a constant velocity 4 m/s, just above the part of the carpet which is still at rest on the floor. Find the speed (in m/s) of the centre of mass of the moving part.



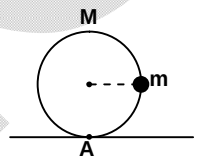
13. Two identical beads each of mass m = 100 gram are connected by an inextensible massless string, can slide along the two arms *AC* and *BC* of a rigid smooth wire frame in a vertical plane. If the system is released from rest, the kinetic energy of the first particle when they have moved by a distance of 0.1 m is $n \times 10^{-3}$ J. Find the value of *n*. (g = 10 m/s^2)



14. Three identical masses are connected with a spring and a string as shown in the figure. They all are initially at rest and the spring at natural length and string is just taut. The maximum extension in the spring is found to be $\frac{nF}{k}$, then find the value of n.

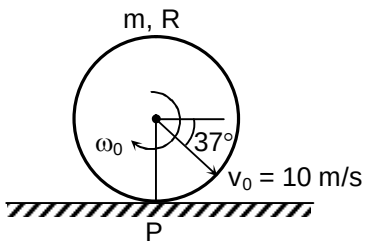


15. A uniform circular disc of mass M and radius *R* has a small mass *m* attached at the edge as shown in the figure. The system is placed on a perfectly rough horizontal surface such that mass *m* is at the same horizontal level as the centre of the disc. It is assumed that there is no slipping at point of contact A. Find the normal reaction (in newton) at point of contact A just after the system is released from rest. (*M* = 2 kg, *m* = 1 kg, *R* = 1m, g = 10 m/s^2)



16. Two balls of unequal masses, moving in opposite directions with equal speeds collide elastically. After collision, the heavier ball is observed to be deviated from its original direction of motion by an angle 30° in the ground frame and by an angle 60° in the centre of mass frame. Find the ratio of mass of heavier ball to that of lighter ball.

17. A ball of mass m = 4 kg and radius R = 0.5 m having initial angular velocity ω_0 = 30 rad/s and initial velocity v_0 = 10 m/s collides with a rough horizontal surface with e = 0.5 as shown in the figure. The coefficient of friction between the ball and surface is μ = 0.5. Find the impulse (in N-s) on the ball due to friction during the collision.



Chemistry

PART – II

SECTION – A
(One or More than one correct type)

*This section contains **THREE (03)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

18. The **correct** statement(s) related to the compounds of Boron is(are)
(A) Orthoboric acid is a protonic acid
(B) Diborane on hydrolysis produces metaboric acid
(C) Borazine reacts with HCl to produce $B_3N_3H_9Cl_3$
(D) Boric acid behaves as a strong acid in the presence of glycol

19. The **correct** statement(s) related to the compounds of alkaline earth metals is(are)
(A) In the solid state Beryllium chloride has a polymeric structure.
(B) Lattice energy of $BaSO_4$ is more than it's hydration energy.
(C) Magnesium carbide on hydrolysis produces acetylene.
(D) $Ca(HCO_3)_2$ does not exist in the solid state.

20. The **incorrect** statement(s) among the following statements is(are)
(A) In trisilylamine molecule the nitrogen atom is sp^2 hybridized.
(B) Al_2Cl_6 has three-centre two electron bonds.
(C) In PF_2Cl_3 molecule the P – F axial bond length is greater than P – Cl equatorial bond length.
(D) Bond angle in NF_3 is greater than the bond angle in NH_3.

SECTION – A
(One Options Correct Type)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.*

21. The radial probability distribution curve of an orbital of hydrogen has 3 local maxima and the orbital has 2 angular node, the orbital will be
(A) 4p (B) 5d
(C) 5f (D) 7s

22. An amount of solid NH_4HS is placed in a closed flask already containing $NH_3(g)$ at certain temperature and 0.4 atm pressure. NH_4HS decomposes to yield $NH_3(g)$ and $H_2S(g)$ in the flask. At equilibrium, the total pressure in the flask was found to be 0.64 atm. The equilibrium constant (K_P) for the equilibrium
$NH_4HS(s) \rightleftharpoons NH_3(g) + H_2S(g)$ is
(A) 4.096×10^{-1} (B) 1.024×10^{-3}
(C) 1.6×10^{-1} (D) 6.24×10^{-2}

23. 10 ml of $\frac{M}{10}$ NH_4OH solution is treated with 4 ml $\frac{M}{10}$ H_2SO_4 solution. The pH of the resulting solution will be
$[pK_b\ NH_4OH = 4.76, \log 2 = 0.3]$
(A) 5.36 (B) 8.94
(C) 8.64 (D) 5.06

24. The **incorrect** statement among the following statement is
(A) The mixture of CO(g) and H_2(g) is known as water gas.
(B) The mixture of CO(g) and N_2(g) is known as producer gas.
(C) Sn^{2+} ion is more stable than Sn^{4+} ion.
(D) Calcium carbide when heated with N_2(g) at 1100°C produces $CaCN_2$.

SECTION – A
(Matching List Type)

*This section contains **FOUR (04)** Matching List Type Questions. Each question has FOUR statements in **List-I** entries (P), (Q), (R) and (S) and FIVE statements in **List-II** entries (1), (2), (3), (4) and (5). The codes for lists have choices (A), (B), (C), (D) out of which **ONLY ONE** of these four options is correct answer.*

25. Match the type of silicate given in List – I with the formula of the silicate given in List – II and choose correct option:

List – I		List – II	
(P)	Pyrosilicate	(1)	$Ca_3Si_3O_9$
(Q)	Orthosilicate	(2)	$BaAl_2Si_2O_8$
(R)	Cyclic silicate	(3)	$Al_2(OH)_4Si_2O_5$
(S)	Sheet silicate	(4)	$Sc_2Si_2O_7$
		(5)	Zn_2SiO_4

(A) P → 2; Q → 4; R → 3; S → 1
(B) P → 2; Q → 4; R → 1; S → 5
(C) P → 4; Q → 5; R → 1; S → 3
(D) P → 3; Q → 2; R → 5; S → 1

26. Match the molecules in List – I with their magnetic property and bond order respectively in List – II and choose the correct option

List – I		List – II	
(P)	C_2	(1)	Diamagnetic , 3
(Q)	N_2	(2)	Paramagnetic, 1
(R)	B_2	(3)	Diamagnetic , 1
(S)	O_2	(4)	Diamagnetic , 2
		(5)	Paramagnetic, 2

(A) P → 3; Q → 2; R → 5; S → 1
(B) P → 5; Q → 3; R → 4; S → 1
(C) P → 4; Q → 1; R → 5; S → 2
(D) P → 4; Q → 1; R → 2; S → 5

27. Match the order of reaction in List – I with the expression for rate constant in List – II and choose the correct option
a = initial concentration of reactant
a – x = concentration of reactant at time 't'

List – I		List – II	
(P)	First order reaction	(1)	$k = \frac{x}{t}$
(Q)	Second order reaction	(2)	$k = \frac{2}{t}\left[\frac{1}{(a-x)^2} - \frac{1}{a^2}\right]$
(R)	Third order reaction	(3)	$k = \frac{2.303}{t}\log\frac{a}{a-x}$

(S)	Zero order reaction	(4)	$k=\frac{1}{t}\left[\frac{1}{a-x}-\frac{1}{a}\right]$
		(5)	$k=\frac{1}{2t}\left[\frac{1}{(a-x)^2}-\frac{1}{a^2}\right]$

(A) P → 1; Q → 4; R → 2; S → 3
(B) P → 3; Q → 5; R → 2; S → 1
(C) P → 3; Q → 4; R → 5; S → 1
(D) P → 3; Q → 2; R → 4; S → 1

28. Match the molecule/ion in the List – I with the bond pair (BP) and lone pair (LP) of electron in the central atom and choose the correct option

List – I		List – II	
(P)	SF_3Cl	(1)	BP = 2, LP = 3
(Q)	ICl_2^-	(2)	BP = 4, LP = 2
(R)	XeF_5^+	(3)	BP = 4, LP = 1
(S)	BrF_3	(4)	BP = 3, LP = 2
		(5)	BP = 5, LP = 1

(A) P → 1; Q → 5; R → 3; S → 2
(B) P → 3; Q → 1; R → 5; S → 4
(C) P → 3; Q → 5; R → 1; S → 4
(D) P → 4; Q → 1; R → 2; S → 5

SECTION – B

(Numerical Answer Type)

*This section contains **SIX (06)** Numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.*

29. For the redox reaction
$xMnO_4^- + yC_2O_4^{2-} + zH^+ \longrightarrow aMn^{2+} + bCO_2 + cH_2O$
the value of x + y + b is_____

30. The total number of lone pair of electrons in XeF_4 is_____

31. The speed of an electron in an orbit of hydrogen atom is 4. 36 × 10^5 m/sec. Total number of waves formed by the electron in one complete revolution in this orbit is_____

32. At 300 K, NH_3 (g) was taken in a flask at a pressure of 2.5 bar. Flask was heated at constant volume to a temperature of 600 K, NH_3 (g) gets partially dissociated to N_2 (g) and H_2 (g) with a degree of dissociation 40%. If at 300 K, NH_3 (g) was undissociated, final pressure of the flask would be_____bar

33. The number of moles of CH_3COONa added to 1 litre of 0.2 M CH_3COOH solution to get a solution of pH = 4 is $x\times10^{-2}$. The value of x is_____
$\left(K_a \text{ of } CH_3COOH = 2\times10^{-5}\right)$

34. 40 ml of 0.2 M NH_4Cl solution was added to 'x' ml of 0. 2 M NH_4OH solution and the pH of the resulting was found to be 8.56. The value of 'x' is_____
$\left(pK_b\ NH_4OH = 4.74, \log 5 = 0.7\right)$

Mathematics

PART – III

SECTION – A
(One or More than one correct type)

*This section contains **THREE (03)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

35. Given a function $f(x) = x^2$, define $g_1(x) = f(x)$ and $g_{(n+1)}(x) = \min\limits_{0\le t\le x}\left(g_n(t) + f(x-t)\right)$, where $n \in N$, then

(A) $\sum\limits_{n=1}^{2024} g_n\left(\frac{1}{\sqrt{n+1}}\right) = \frac{2023}{2024}$

(B) $\sum\limits_{n=1}^{2024} g_n\left(\frac{1}{\sqrt{n+1}}\right) = \frac{2024}{2025}$

(C) $\sum\limits_{n=2}^{2024} g_n\left(\frac{1}{\sqrt{n-1}}\right) = \frac{2023}{2024}$

(D) $\sum\limits_{n=2}^{2024} g_n\left(\frac{1}{\sqrt{n-1}}\right) = \frac{2024}{2023}$

36. The curve $y = tx^2 + tx + \frac{1}{48}$ and $x = ty^2 + ty + \frac{1}{48}$ $(t \neq 0)$ touch each other for

(A) two rational values of t
(B) two irrational values of t
(C) four rational values of t
(D) four irrational values of t

37. If $S_m = \int\limits_{-m(m!)\pi}^{m(m!)\pi} |\sin x|\left[\sin x\right]dx$ (where [.] represents greatest integer function) and $R_n = \sum\limits_{m=1}^{n} S_m$, then

(A) $R_n < 0\ \forall\ n \in N$

(B) $\frac{S_m}{S_{m+1}} \le \frac{1}{4}\ \forall\ m \in N$

(C) $\lim\limits_{n\to\infty} \frac{R_n}{(n+1)!} = -2$

(D) $R_n = (n+1)!$

SECTION – A
(One Options Correct Type)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.*

38. If $f : (0, \infty) \to (0, \infty)$ satisfy $f(xf(y)) = x^3y^a$ $(a \in R)$, then $\sum\limits_{r=1}^{n} f(r)\cdot {}^nC_r$ is equal to

(A) $n(n+3)2^{n-3}$

(B) $n(n-3)2^{n+3}$

(C) $n^2(n+3)2^{n-3}$

(D) $n^2(n-3)2^{n+3}$

39. The ends of a rod of length 4 unit slide inside a parabolic track $y = x^2$, indefinitely in both direction. As they do, midpoint of rod traces a curve, then area of region bounded between the parabola and traced curve is

(A) 2π
(B) 4π
(C) 6π
(D) 8π

40. $\int \frac{\sqrt{2\sin 2x + 4\cos^2 x + \sin 4x} - 2x}{1+\sin 2x} dx$

(A) $\frac{2x\sin x}{(\sin x + \cos x)^2} + c$

(B) $\frac{x\cos x}{\sqrt{1+\sin 2x}} + c$

(C) $\frac{2x\cos x}{\sin x + \cos x} + c$

(D) $\frac{x\sin x}{\sqrt{1+\sin 2x}} + c$

41. Consider the integral $I_1 = \int_1^e (1+x)(x+\ell nx)^{2022} dx$ and $I_2 = \int_{\sin^{-1}\left(\frac{1}{e}\right)}^{\frac{\pi}{2}} (1+e\sin x + \ln\sin x)^{2023} \cos x\, dx$, then $I_1 + \frac{eI_2}{2023}$ is equal to

(A) $\frac{(e+1)^{2023}}{2023}$

(B) $\frac{e(e+1)^{2023} - 1}{2023}$

(C) $\frac{(e+1)^{2022}}{2022}$

(D) $\frac{e(e+1)^{2022} - 1}{2022}$

SECTION – A
(Matching List Type)

*This section contains **FOUR (04)** Matching List Type Questions. Each question has FOUR statements in **List-I** entries (P), (Q), (R) and (S) and FIVE statements in **List-II** entries (1), (2), (3), (4) and (5). The codes for lists have choices (A), (B), (C), (D) out of which **ONLY ONE** of these four options is correct answer.*

42. Match the following List-I with List-II

LIST – I		LIST - II	
(P)	The shortest distance between the curves $2y^2 = x^3$ and $9x^2 + 9y^2 - 60y + 91 = 0$ is	(1)	$\frac{11\sqrt{3}}{6}$
(Q)	The shortest length of chord intercepted on a normal to the curve $x^2 - 2x - 4y + 9 = 0$	(2)	$\frac{1}{\sqrt{3}}$
(R)	The maximum value of the expression $\tan\left(x+\frac{2\pi}{3}\right) - \tan\left(x+\frac{\pi}{6}\right) + \cos\left(x+\frac{\pi}{6}\right)$ in the interval $\left[-\frac{5\pi}{12}, -\frac{\pi}{3}\right]$ is	(3)	$\frac{\sqrt{52}-3}{3}$
(S)	A circle of radius 2 unit touches positive x-axis and positive y-axis at P and Q respectively. A variable line L passing through origin intersect circle C in two points M and N. Find slope of line L for which area of triangle MNQ is maximum	(4)	$5+\sqrt{2}$
		(5)	$6\sqrt{3}$

The correct option is:

(A) (P) → (3); (Q) → (5); (R) → (1); (S) → (4)

(B) (Q) → (4); (P) → (3); (R) → (2); (S) → (1)

(C) (R) → (4); (Q) → (3); (P) → (1); (S) → (5)

(D) (P) → (3); (Q) → (5); (R) → (1); (S) → (2)

43. Match the following List-I with List-II

LIST – I		LIST - II	
(P)	A continuous and differentiable function f(x) satisfies the functional equation $f(x+y)=\frac{f(x)f(y)}{f(x)+f(y)}$; f(1) = 16, then f(4) equals	(1)	$\frac{7}{12}$
(Q)	Let f : R → R be a differentiable function satisfying $f\left(\frac{x+y}{5}\right)=\frac{6+f(x)+f(y)}{5}$ ∀ x, y ∈ R and f′(2) = 2, then area bounded by relation |y| = h(x), x = 2 and x = –2 where h(x) = |f(|x|) – 4| is	(2)	4
(R)	Let f be a differentiable function satisfying the relation $f(x+2y)=f(x)+f(2y)+e^{x+2y}(x+2y)-xe^{x}-2ye^{2y}+4xy$ ∀ x, y ∈ R and f′(0) = 1, then the value of $\lim_{x\to0}\frac{f(2x)-4x^2-2x}{x^2}$ is	(3)	2
(S)	Let f(x) is a differentiable function satisfying $f(y)-f(x)=\frac{x^x}{y^y}f\left(\frac{y^y}{x^x}\right)$ ∀ x, y ∈ R^+, if f′(1) = 1, then area bounded between the curve f(x) and 9(x) = 2x – $2x^2$ is	(4)	8
		(5)	$\frac{7}{16}$

The correct option is:
(A) (P) → (2); (Q) → (3); (R) → (4); (S) → (1)
(B) (P) → (2); (Q) → (4); (R) → (2); (S) → (1)
(C) (P) → (4); (Q) → (3); (R) → (1); (S) → (5)
(D) (P) → (4); (Q) → (3); (R) → (2); (S) → (5)

44. Match the following List-I with List-II

LIST – I		LIST - II	
(P)	The general solution of differential equation $y(xy-(x^2-y^2)^2)\,dx=(y^3-x(x^2-y^2)^2)\,dy$ is	(1)	$-\cot\left(x^2+y^2\right)=\left(\frac{x}{y}\right)^2+c$
(Q)	The solution of the differential equation $(xy^4+y)\,dx-x\,dy=0$ is	(2)	$\frac{2x}{y}+\frac{1}{x^2-y^2}=c$
(R)	Solution of the differential equation $y(2x^4+y)\,dy=(1-4xy^2)\,x^2\,dx$ is	(3)	$\frac{x^4}{4}+\frac{1}{3}\left(\frac{x}{y}\right)^3=c$
(S)	The solution of differentiable equation $(1+xy)\,ydx+x(1-xy)\,dy=0$ is	(4)	$3(x^2y)^2+y^3-x^3=c$
		(5)	$\ln\left(\frac{x}{y}\right)-\frac{1}{xy}=c$

The correct option is:
(A) (P) → (2); (Q) → (3); (R) → (4); (S) → (1)
(B) (Q) → (4); (P) → (3); (R) → (2); (S) → (5)
(C) (P) → (2); (Q) → (3); (R) → (4); (S) → (5)
(D) (Q) → (4); (P) → (3); (R) → (2); (S) → (1)

45. Match the following List-I with List-II

LIST – I		LIST - II	
(P)	Let $f(x) = x^2 \sin\frac{\pi}{x}$ $(x \neq 0)$, then the number of points in $\left(\frac{1}{21}, 1\right)$ at which $f'(x) = 0$ is	(1)	18
(Q)	Let $f(x) = x^2 + px + q + 3$, if $f(f(1)) = f(f(-2)) = 0$ where $f(1) \neq f(-2)$ and $g(x) = x^2 + (8p^2 - 18p)x - q + 2$, then $2g(2)$ is	(2)	20
(R)	Let $f(x) = x^9 - 3x^8 - 53x^7 - 8x^6 - 8x^5 - 8x^4 - 8x^3 - 8x^2 - 8x + 9$, then $f(9)$ will be equal to	(3)	23
(S)	Let $L = \lim\limits_{x\to 0}\left(1 + e^{-\frac{1}{x^2}} \cdot \tan^{-1}\frac{1}{x^2} + x^2 \cdot e^{-\frac{1}{x^2}} \cdot \sin\frac{1}{x^4}\right)^{e^{\left(\frac{1}{x^2}\right)}}$, then $\left[15\log_e L\right]$ is (Where [.] represents greatest integer function)	(4)	27
		(5)	32

The correct option is:
(A) (P) $\to$ (2); (Q) $\to$ (4); (R) $\to$ (1); (S) $\to$ (3)
(B) (Q) $\to$ (4); (P) $\to$ (3); (R) $\to$ (2); (S) $\to$ (1)
(C) (R) $\to$ (4); (Q) $\to$ (3); (P) $\to$ (1); (S) $\to$ (5)
(D) (P) $\to$ (2); (Q) $\to$ (4); (R) $\to$ (5); (S) $\to$ (3)

SECTION – B

(Numerical Answer Type)

*This section contains **SIX (06)** Numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.*

46. Let $I_1 = \lim\limits_{x\to\infty} x\left(2x - \sqrt[3]{x^3 + x^2 + 1} - \sqrt[3]{x^3 - x^2 + 1}\right)$ and $I_2 = \lim\limits_{x\to 0} \frac{e^{x\tan(2025)x} - 1}{x\ln(1+x)}$, then the value of $I_1 I_2$ is equal to

47. If $f : R \to R$ is a monotonic differentiable real valued function and a, b are two real numbers such that $\int\limits_a^b \left(f(x) + f(a)\right)\left(f(x) - f(a)\right)dx = k\int\limits_{f(a)}^{f(b)} x\left(b - f^{-1}(x)\right)dx$, then the value of k is

48. If $I_1 = \int\limits_0^1 x^{\frac{5}{2}}(1-x)^{\frac{7}{2}}dx$, $I_2 = \int\limits_0^1 \frac{x^{\frac{5}{2}}(1-x)^{\frac{7}{2}}}{(7+x)^8}dx$ and $I_1 = 7(n)^{\frac{7}{2}} \cdot I_2$ where $n \in N$, then the value of n is

49. If the equation $\sqrt{x^2 + 2ax} = \sqrt{4x - 4a - 13}$ has only one solution, then the number of integral value of a is

50. If $f(x) = \frac{x^{39}}{17} - 10x^{16} + 7$ and $g(x) = 39 \cdot x^{38} - x^5 + 1$, then $\lim\limits_{x\to\infty} g(x) \cdot \int\limits_0^x e^{f(t)-f(x)}dt$ is

51. Let $f(x) = \tan^{-1}\left(\frac{\left(\sqrt{12}-2\right)x^2}{x^4+2x^2+3}\right)$ and m and M are respectively minimum and maximum values of f(x) and x = a, (a > 0) is in the domain of f(x) where f(x) attains its maximum value, then the value of $\left|4\tan\left(a^4\left(\sec^{-1}\left(\frac{2}{a^2}\right)+M+m\right)\right)\right|$ is equal to